Comparison between Full Order and Minimum Order Observer Controller for DC Motor

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Abstract

This paper presents comparison between full order and minimum order observer controller design using MATLAB environment. As a matter of fact, it has shown how to implement a dc motor state space model and its observer. Different observer gain(s) are determined by selecting different Eigen values for the observers. Both designed controllers are tested using root locus method by varying different observer poles. Having full understanding of the observer controller implementation(s), students and engineers will fill confident to use this observers and observer based controllers in numerous engineering and scientific applications.

Key words: Observer controller Design, State space, DC Motor, Minimum-Order Observer, Implementation in MATLAB.

1. Introduction

In this paper we shall consider a problem of designing a regulator systems by using the the pole-placement-with -observer approach. We shall use the following design procedure.

1. Derive a state space model of a plant. In this paper a DC motor is considered as a plant and armature current and speed of the dc motor are taken as state variables.
2. Chose the desired closed-loop poles for pole placement and chose the desired observer poles.
3. Determine the state feedback gain matrix K and the observer gain matrix Ke.
4. Using gain matrix K and Ke obtained in step 3, derive the transfer function of the observer controller. Root Locus methd is used to analyse the stability of the controller based on location of the observer poles.

Observers are used these days not only for the purpose of feedback control, but also in their own right to observe state variables of a dynamic system, which can be an experiment in progress whose state has to be monitored at all times. However, in the most recent editions of several standard undergraduate control system textbooks we can find the coverage of full-order and even reduced-order observers [2]-[4].

2. DC Motor Modeling Using State Space Analysis

The different equations related to DC motor are given below [11],

$$e_m(t) = K_m \frac{d\theta(t)}{dt} \quad (1)$$

$$e_a(t) = L_m \frac{di_a(t)}{dt} + R_m i_a(t) + e_m(t) \quad (2)$$

$$T(t) = K_t i_a(t) \quad (3)$$

$$J\frac{d\theta^2(t)}{dt^2} + B\frac{d\theta(t)}{dt} = T(t) \quad (4)$$

Where $e_a(t)$ = armature voltage, $e_m(t)$ = back emf, $i_a(t)$ = armature current, $T(t)$ = developed torque, $\theta(t)$ = motor shaft angle, $\frac{d\theta(t)}{dt} = \omega(t)$ = shaft speed, J = moment of inertia of the rotor, B = viscous frictional constant, L_m = inductance of armature windings, R_m = armature winding resistance, K_t = motor torque constant, K_m = motor constant.

Here the motor speed ω(t) is controlled by varying the armature voltage $e_a(t)$. Hence $e_a(t)$ is the input variable and ω(t) is the output variable.

We chose as the state variables $x_1(t) = \omega(t) = \frac{d\theta(t)}{dt}$ and

$$x_2(t) = i_a(t) \quad (5)$$

The state equations will now be derived by using above equations.

$$\frac{dx_1(t)}{dt} = -\frac{B}{J}x_1(t) + \frac{K_t}{J}x_2(t) \quad (6)$$

$$\frac{dx_2(t)}{dt} = -\frac{K_m}{L_m}x_1(t) - \frac{R_m}{L_m}x_2(t) + \frac{1}{L_m}e_a \quad (7)$$

$$y(t) = \frac{d\theta(t)}{dt} = \omega(t) = x_1(t) \quad (8)$$

Hence state model of dc motor is derived from equations (6), (7) and (8) as follows

$$\begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{B}{J} & \frac{K_t}{J} \\ -\frac{K_m}{L_m} & -\frac{R_m}{L_m} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L_m} \end{bmatrix} u\ (t) \quad (9)$$

$$y(t) = [1 \quad 0] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad (10)$$

3. DC Motor State Model Using Motor Parameters

Let, the motor parameters (coefficient of differential equations) are assigned to be $L_m = 0.5$ H, K_t = 0.01 N-m/A, K_m= 0.01 V-sec/rad, J = 0.01 kg-m2, B = 0.1 N-m-sec/rad, R_m= 1 Ω.

Thus the state model of dc motor is derived using motor parameters and equation (9) and (10) as follows:

$$\begin{bmatrix} \frac{dx_1(t)}{dt} \\ \frac{dx_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} -10 & 1 \\ -0.02 & -2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u(t) \quad (11)$$

$$y(t) = [1 \quad 0] \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad (12)$$

4. Full order observer design

The theory of observers originated in the work of Luenberger in the middle of1960s, [7]-[9].According to Luenberger, any system driven by the output of the given system can serve as an observer for that system. Consider a linear dynamic system with unknown initial value of x

$$\dot{x} = Ax + Bu\ , y = Cx \quad (13)$$

We define the mathematical model of the observer to be

$$\dot{\tilde{x}} = (A–KeC)\tilde{x} + Bu + Key \quad (14)$$

Where $\tilde{x}$ is the estimated state and C $\tilde{x}$ is the estimated output. The n×1 matrix Ke is called state observer gain matrix. Hence the observer error equation is defined by

$$\dot{x} - \dot{\tilde{x}} = (A – KeC)\ (x - \tilde{x})$$

$$\dot{e} = (A–KeC)\ e \quad (15)$$

Where (x - $\tilde{x}$) = e = observer estimation error vector. Thus the dynamic behavior of the error vector depends upon the Eigen values of A – KeC .

If the observer gain Ke is chosen such that the feedback matrix A–KeC is asymptotically stable (has all eigen values with negative real parts) then the estimation error e will decay to zero for any initial condition of e. This stabilization requirement can be achieved if pair (A,C) is observable.

The system (14) under the perfect state feedback control, that is

$u = -Kx$

has the closed loop form

$$\dot{x} = (A - BK)x \quad (16)$$

So the eigen values of the matrix $A - BK$ are the closed loop eigen values under perfect state feedback.

For state feedback control $\tilde{x}$, It is known that

$u = -K\tilde{x}$

With this control, the state equation (13) becomes

$$\begin{aligned} \dot{x} &= Ax - BK\tilde{x} \\ &= (A - BK)x + BK(x - \tilde{x}) \\ &= (A - BK)x + BKe \end{aligned} \quad (17)$$

Where x -$\tilde{x}$ = e = observer estimation error

Combining equations (17) and (15), we obtain

$$\begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A - BK & BK \\ 0 & A - K_e C \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} \quad (18)$$

The above equation describes the dynamic of the observed state feedback control.

Ackerman's formula is used to write the program in MATLAB command window. The programs are given below [12].

```
>> %State feedback matrix K design using pole-placement technique

A = [-10 1;-0.02 -2];
B = [0; 2]; C = [1 0]; D = [0];
J = [-2+j*1 -2-j*1];
K = acker (A,B,J)

K =

  32.4900   -4.0000

>> %State observer gain matrix Ke design

>> A = [-10 1;-0.02 -2];
B = [0;2]; C = [1 0]; D = [0];
L= [-9 -10];
```

Ke = acker (A', C', L)'

Ke =

7.0000
55.9800

5. Full Order Observer Based Controller Design for DC Motor

The transfer function of the observer controller is given as

$$\frac{U(s)}{-Y(s)} = \frac{num}{den} = K(SI - A - K_e C + BK)^{-1} K_e \quad (19)$$

The above transfer function can be represented by following block diagram model.

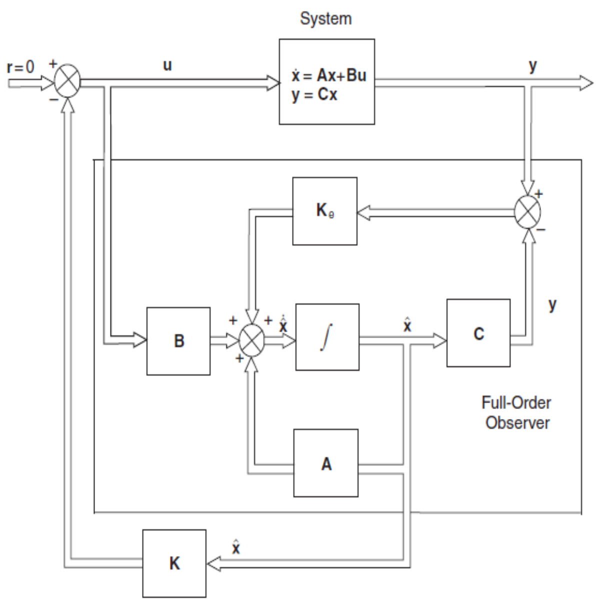

Fig1: Block diagram of system with observed state feedback

The following MATLAB program is used to produce the transfer function of the full order observer controller [12].

```
>> A=[-10 1;-0.02 -2];
B=[0;2];C=[1 0];D=[0];
K=[32.5 -4];
Ke=[7;56];
AA=A-Ke*C-B*K;
BB=Ke;
CC=K;
DD=0;
[num, den]=ss2tf (AA,BB,CC,DD)

num =

   0    3.5000   35.5600

den =

   1.0000   11.0000   19.0200
```

Hence the transfer function of the full order observer controller is

$$\frac{3.5s + 35.56}{s^2 + 11s + 19.02} \quad (20)$$

6. Full Order Observer controller analysis based on Root Locus

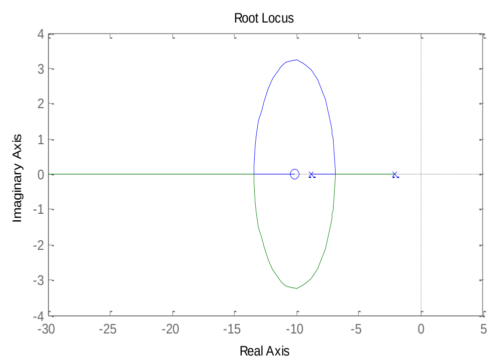

Fig2: The root locus plot of the full order observer controller with observer poles at s =-9 and s =-10.

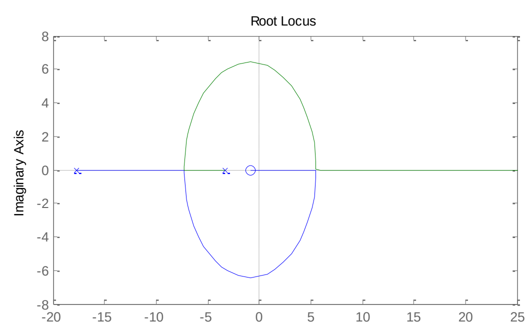

Fig3: The root locus plot of the full order observer controller with observer poles at s =-14 and s =-15

It is seen from the above two figs that If we place the observer poles far to the left of the jw axis, the observer controller become unstable.

7. Minimum Order Observer Design

Consider equation (14) where the state vector X can be partitioned into two parts x_a (a scalar) and x_b (a vector). Here the state variable x_a is equal to the output y and thus can be directly measured and x_b is the unmeasurable portion of the state vector. Then partitioned state and output equations become

$$\begin{bmatrix} \dot{x}_a \\ \dot{x}_b \end{bmatrix} = \begin{bmatrix} A_{aa} & A_{ab} \\ A_{ba} & A_{bb} \end{bmatrix} \begin{bmatrix} x_a \\ x_b \end{bmatrix} + \begin{bmatrix} B_a \\ B_b \end{bmatrix} u \qquad (21)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_a \\ x_b \end{bmatrix} \qquad (22)$$

From equation (15) the equation for the measured portion and unmeasured portion of the state becomes

$$\dot{x}_a = A_{aa} x_a + A_{ab} x_b + B_a u \qquad (23)$$

$$\dot{x}_b = A_{ba} x_a + A_{bb} x_b + B_b u \qquad (24)$$

Here equation (18) and (17) are known as 'state equation' and 'output equation' for the minimum order observer.
The equation for the full order observer is

$$\dot{\tilde{x}} = (A–KeC)\tilde{x} + Bu + Key \qquad (25)$$

Then making the substitution of table 1 into last equation we obtain

$$\dot{\tilde{x}}_b = (A_{bb} - k_e A_{ab}) \tilde{x}_b + A_{ba} x_a + B_b u + k_e A_{ab} x_b \qquad (26)$$

Full order state observer	Minimum order state observer
$\tilde{x}$	$\tilde{x}_b$
A	A_{bb}
Bu	$A_{ba} x_a + B_b u$
y	$\dot{x}_a - A_{aa} x_a - B_a u$
C	A_{ab}
k_e (n×1 matrix)	k_e [(n−1) ×1 matrix]

Table4: List of necessary substitutions for writing the observer equation for the minimum order state observer

By subtracting equation (26) from equation (24) we obtain

$$\dot{x}_b - \dot{\tilde{x}}_b = (A_{bb} - k_e A_{ab}) (x_b - \tilde{x}_b) \qquad (27)$$

Define e = $x_b - \tilde{x}_b$
Then equation (21) becomes

$$\dot{e} = (A_{bb} - k_e A_{ab}) e \qquad (28)$$

This is the error equation for the minimum order observer. The dynamic behavior of error vector depends upon the Eigen values of $A_{bb} - k_e A_{ab}$.
Now the characteristic equation for the minimum order observer is obtained the equation as follows.

$$|SI - A_{bb} + k_e A_{ab}| = (s - m_1)(s - m_2)\ldots(s - m_{n-1}) = 0$$

Where $m_1, m_2, \ldots\ldots m_{n-1}$ are desired Eigen values for the minimum order observer. Suppose the desired location of the Eigen value for the minimum observer is at s = -9.
The minimum observer gain matrix is designed by the following MATLAB program.
Ackerman's formula is used to write the program in command window. The program is given below [13].

```
>> % State observer  gain matrix Ke design.

Aab = [1]; Abb = [-2];
LL = [-9];
>> Ke = acker (Abb', Aab', LL)'
Ke =

    7
```

8. Minimum Order Observer based Controller Design for DC Motor

The transfer function of minimum order observer controller is given as

$$\frac{u(s)}{-y(s)} = \frac{num}{den} = - [\tilde{C} (SI - \tilde{A})^{-1} \tilde{B} + \tilde{D}] \qquad (29)$$

The above transfer function can be represented by following block diagram model.

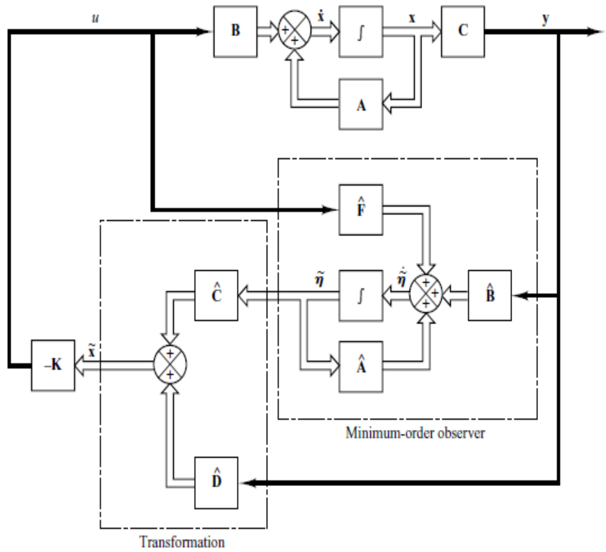

Fig5: System with observed state feedback, where the observer is minimum-order observer

Where $\tilde{A} = \hat{A} - \hat{F}\, k_b$

$\tilde{B} = \hat{B} - \hat{F}\,(k_a + k_b k_e)$

$\tilde{C} = -\, k_b$

$\tilde{D} = -\,(k_a + k_b k_e)$

$\hat{F} = B_b - k_e B_a$

The following MATLAB program is used to produce the transfer function of the minimum observer controller [13]

```
A = [-10 1;-0.02 -2];
B = [0;2]; C = [1 0]; D= [0];
Aab = [1]; Abb= [-2];
Ke = 7; K=[32.5 -4]; Kb=-4;
Ka=32.5;
Aaa = -10; Aba = -0.02;
Ba = 0; Bb=2;
Fhat = Bb-Ke*Ba;
Ahat = Abb-Ke*Aab;
Bhat = Ahat*Ke+Aba-Ke*Aaa;
Atilde = Ahat-Fhat*Kb;
Btilde = Bhat-Fhat*(Ka+Kb*Ke);
Ctilde= -Kb;
Dtilde = - (Ka+Kb*Ke);
[num, den] = ss2tf (Atilde, Btilde,-Ctilde,-Dtilde)

num =

   4.5000   12.5800


den =
    1     1
```

Hence the transfer function of the minimum order observer controller is $\dfrac{4.5\,(S+2.8)}{(S+1)}$ (30)

9. Minimum Order Observer controller analysis based on Root Locus

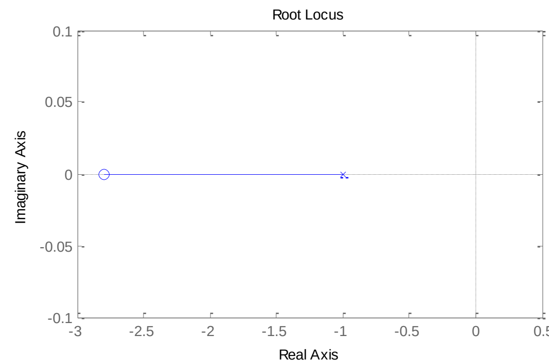

Fig6: The root locus plot of the minimum order observer controller with observer pole at s =-9

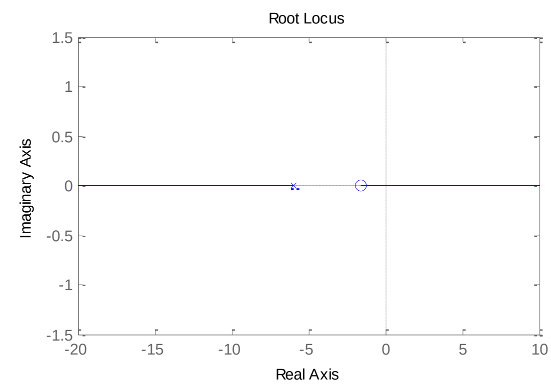

Fig7: The root locus plot of the minimum order observer controller with observer pole at s =-14

It is seen from the above two figs that If we place the observer poles far to the left of the jw axis, the observer controller become unstable

10. Conclusion

This paper has shown in detail how to implement full-order and minimum-order observer controllers based on root locus in MATLAB environment and presented corresponding fundamental derivation and results with the help of dc motor state model and the conclusion is if the observer controller becomes unstable, move the observer poles to the right in the left-half s plane until the observer

controller becomes stable, Also the desired closed-loop pole location may need to be modified.

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